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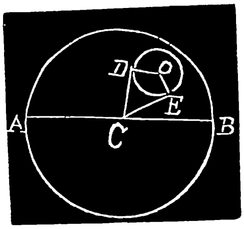
Let C be the center of the semi-circle, O the center of the random circle, CD, CE tangents from C to random circle.



Also let $AC=r$, $CO=x$, $OD=z=OE$, $\angle ACO=\theta$, $\angle DCO=\varphi$, $p=$required chance.

The limits of θ are 0 and $\frac{1}{2}\pi$; of x, 0 and $r/(1+\sin\theta)=x'$, and $r/(1+\sin\theta)=x'$ and r; of z, 0 and $x\sin\theta$, and 0 $r-x$.

Then $p=2\varphi/\pi$, $\varphi=\sin^{-1}(z/x)$.

$$\therefore\ p=\frac{\dfrac{2}{\pi}\displaystyle\int_0^{\frac{1}{2}\pi}\left[\int_{x'}^{r}\int_0^{r-x}\sin^{-1}(z/x)xdxdz+\int_0^{x'}\int_0^{x\sin\theta}\sin^{-1}(z/x)xdxdz\right]d\theta}{\displaystyle\int_0^{\frac{1}{2}\pi}\left[\int_{x'}^{r}\int_0^{r-x}xdxdz+\int_0^{x'}\int_0^{x\sin\theta}xdxdz\right]d\theta}$$

$$=\frac{72}{\pi r^3(3\pi-4)}\int_0^{\frac{1}{2}\pi}\left[\int_{x'}^{r}\int_0^{r-x}\sin^{-1}(z/x)xdxdz+\int_0^{x'}\int_0^{x\sin\theta}\sin^{-1}(z/x)xdxdz\right]d\theta$$

$$=\frac{72}{\pi r^3(3\pi-4)}\int_0^{\frac{1}{2}\pi}\left[\int_{x'}^{r}[(r-x)\sin^{-1}(\frac{r-x}{x})+\sqrt{(2rx-r^2)}-x]xdx\right.$$

$$\left.+\int_0^{x'}(\theta\sin\theta+\cos\theta-1)x^2dx\right]d\theta$$

$$=\frac{4}{\pi(3\pi-4)}\int_0^{\frac{1}{2}\pi}\left[2-\frac{3\theta}{(1+\sin\theta)^2}-\frac{(2+3\sin\theta+\sin^2\theta)\cos\theta}{(1+\sin\theta)^3}\right]d\theta$$

$$=\frac{4}{\pi(3\pi-4)}(\pi-\tfrac{1}{4}-2\log2)=\frac{1}{3\pi-4}(4-\frac{1}{\pi}-\frac{8}{\pi}\log2).$$

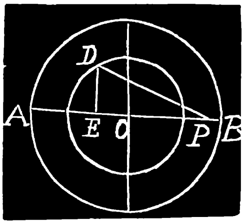
104. Proposed by LON C. WALKER, A. M., Assistant Professor of Mathematics, Leland Stanford Jr. University, Palo Alto, Cal.

In a given sphere two radii are drawn at random, and a point taken in each at random. (1) Find the chance that the distance between the two points does not exceed the radius of the sphere. (2) Find the distance between them.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let P be one of the points on radius OB, D the second point on radius OD. Let $OB=r$, $OP=x$, $OC=OD=y$, $OE=z$.

Then $PD=\sqrt{(x^2+y^2+2xz)}$.



The first point lies on the surface of a sphere radius x. An element of surface of this sphere is $2\pi xdx$. The second point lies on the surface of the sphere radius y. An element of surface of this sphere is $2\pi ydy$.

Let $(r^2-x^2-y^2)/2x=z'$, $p=$chance, and $\Delta=$the average distance. Then

$$(1).\ p=\frac{1}{(\frac{4}{3}\pi r^3)\pi r^2}\int_0^r\int_0^r\int_{-y}^{z'}2\pi x dx.2\pi y dy dz=\frac{3}{2r^5}\int_0^r\int_0^r y[r^2-(x-y)^2]dxdy$$

$$=\frac{1}{8r^5}\int_0^r(3r^4-6r^2x^2+8r^3x)dx=\tfrac{5}{8}.$$

$$(2).\ \Delta=\frac{1}{(\frac{4}{3}\pi r^3)\pi r^2}\int_0^r 2\pi x dx\Big[\int_0^x\int_{-y}^{y}2\pi y\sqrt{(x^2+y^2+2xz)}\,dydz$$

$$+\int_x^r\int_{-y}^{y}2\pi y\sqrt{(x^2+y^2+2xz)}\,dydz\Big].$$

$$\therefore\ \Delta=\frac{1}{r^5}\int_0^r dx\Big[\int_0^x y[(x+y)^3-(x-y)^3]dy+\int_x^r y[(y+x)^3-(y-x)^3]dy\Big]$$

$$=\frac{1}{10r^5}\int_0^r(15r^4x+10r^2x^3-x^4)dx=\tfrac{59}{60}r.$$

Also solved by the *PROPOSER*.

105. Proposed by L. C. WALKER, Assistant in Mathematics, Leland Stanford Jr. University, Palo Alto, Cal.

Find the average distance of the center of an ellipsoid, axes $2a$, $2b$, and $2c$ from its surface.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Different interpretations will result in different solutions. The method generally employed by the best thinkers in this department of mathematics is the following:

$$d=\frac{\int r ds}{\int ds}$$, where ds is an element of surface.

Let s be the eighth part of the surface of the ellipsoid.

Then $d=\frac{1}{s}\int r ds$.

$$ds=\sqrt{1+\left(\frac{dz}{dx}\right)^2+\left(\frac{dz}{dy}\right)^2}dxdy=\left[\frac{a^4b^4-b^4(a^2-c^2)x^2-a^4(b^2-c^2)y^2}{a^2b^2(a^2b^2-b^2x^2-a^2y^2)}\right]^{\frac{1}{2}}dxdy$$

$$r=\sqrt{(x^2+y^2+z^2)}=\frac{1}{ab}[a^2b^2c^2+b^2(a^2-c^2)x^2+a^2(b^2-c^2)y^2]^{\frac{1}{2}}.$$

$$\therefore\ p=\int_0^a\int_0^{(b/a)\sqrt{(a^2-x^2)}}[c^2+x^2+y^2-\frac{c^2}{b^4}(b^2-c^2)y^2-\frac{c^2}{a^4}(a^2-c^2)x^2$$